2026 Form A Solutions

Clover Math Competition

May 1, 2026

1. We basically want to find the value of $\frac{2026}{45}$, then round down because that is the maximum number of full copies that Katelyn can make. Because $\frac{2026}{45} = 45 + \frac{1}{45}$, the answer is $\boxed{45}$.

2. When the coin is flipped two times, we can consider each flip separately. The probability that it lands tails the first time is $\frac{2}{3}$, and the probability that it lands tails the second time is $\frac{2}{3}$ as well. Because we want that to happen both times, the answer is $\frac{2}{3} \cdot \frac{2}{3} = \boxed{\frac{4}{9}}$.

3. We can use the approximation $\pi \approx 3.14$, from which $5\pi \approx 15.7$. Then because $16 = 4^2$ is so close to 15.7 and the next closest term is $3^2 = 9$, then the answer is clearly $\boxed{4}$.

   Remark: It suffices to know that $3 < \pi < 4$ to solve this problem by showing both $\sqrt{15}$ and $\sqrt{20}$ round to 4, although it would certainly be easier to know the approximation above.

4. The greatest common divisor is at most 6, because 20260 and 20266 differ by 6. But none of them are divisible by 3, which can be verified by the 3 divisibility rule. But they are divisible by 2, so the answer is just $\boxed{2}$.

5. Let the price of a laptop be L, and let the price of a phone be P. We are given that $L = 1.5P$, and are trying to find how many Ps would equal 60% greater than $15L$. But that can be expressed as

$$1.6 \cdot 15L = 8 \cdot 3L = 24L = 24(1.5P) = 36P.$$

   From this, the answer follows as $\boxed{36}$.

6. Suppose those two angles mentioned are x and $x + 120$ degrees. Because the triangle is isosceles, the third angle must either equal x or $x + 120$.

   Case 1: The third angle equals x

   Then $x + (x + 120) + x = 180 \implies x = 20$, which would make the larger angle $120 + 20 = 140°$.

   Case 2: The third angle equals $x + 120$

   Then $x + (x + 120) + (x + 120) = 180 \implies x = -20$. But because one of the angles equals x, then we would obtain a negative angle, which is clearly wrong. So this doesn't yield any triangle.

   Therefore, the only possible degree measure of the larger angle is $\boxed{140}$.

7. This problem just boils down to testing each case.

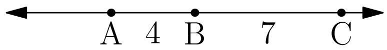
   If Betty is a knight, then she would've told the truth and said she was a knight, contradiction. So she is not a knight.

   Similarly, if Betty was a knave, then she would lie and say she is either a knight or a joker. So this is not possible either.

   Finally, our work has narrowed down Betty to being a joker. And indeed this works, since she could tell a lie and say she is a knave. Therefore, only the third statement can possibly be true, making the answer $\boxed{4}$.

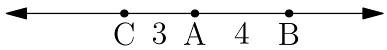
8. Place Abel and Beth first on a line in that order. Note that this can be done because had they been placed the other way around, we could just reflect the diagram over and it wouldn't change their distances. We will perform casework on which direction Charlie is placed relative to Beth.

Case 1: Charlie is to the right of Beth



Then we obtain the situation shown. It remains to place Darelyn into the diagram. If Darelyn is to the right of Charlie, then she would be 16 houses away from Abel. If Darelyn is to the left of Charlie, then she would be 6 houses away from Abel. Neither scenario fits the 8 houses given, so this case cannot happen.

Case 2: Charlie is to the left of Beth



Then we obtain the situation shown. Here, Charlie is 3 to the left of Abel. If Darelyn is to the right of Charlie, then she would be 2 houses away from Abel, which doesn't work. However, if Darelyn is to the left of Charlie, then indeed, she would be 8 houses away from Abel. So this scenario would work, and here Abel would be $\boxed{3}$ houses away from Charlie.

Remark: This "puzzle" problem was inspired from deciphering linear order of genes on a chromosome using given recombination frequencies.

9. First consider the conditions that remainder when it is divided by 5 is 3, and the remainder when it is divided by 3 is 1. The smallest positive integer satisfying those conditions can be found as 13 by listing out 3, 8, and 13 from the first condition. Next, we want the positive integer to be even, but observe that adding $\text{lcm}(3, 5) = 15$ (which preserves the remainders from 3 and 5) already gives our answer, $\boxed{28}$.

Alternate Solution: This problem can be completed slightly faster using modular arithmetic. We are essentially given the system shown below, and want to find the smallest positive integer n satisfying them.

$$\begin{cases} n \equiv 0 \pmod 2 \\ n \equiv 1 \pmod 3 \\ n \equiv 3 \pmod 5 \end{cases}$$

However, observe that each remainder is two less than the divisor. So we can rewrite the system as

$$\begin{cases} n \equiv -2 \pmod 2 \\ n \equiv -2 \pmod 3 \\ n \equiv -2 \pmod 5 \end{cases} \implies n \equiv -2 \pmod{30}.$$

From this our answer would quickly follow to be $\boxed{28}$.

10. In the first half, Timmy had caught 3 of the 12 balls. Suppose in the second half that he caught x balls, so that he was thrown $2x$ balls. Then the total number he caught is $x + 3$, compared to the total thrown at him which was $2x + 12$. Looking for a 44% catch rate, then we get the equation

$$\frac{x+3}{2x+12} = \frac{11}{25}.$$

Cross multiplying yields $25x + 75 = 22x + 132$, from which we can find $3x = 57 \implies x = 19$. But we are trying to find the total number of balls he caught in the game, so we need to add back the first half's three caught balls too. Then our answer is $19 + 3 = \boxed{22}$.

11. We can first choose which digit will be the 0 before selecting the rest of the digits. Treating them as four "slots", the leftmost digit cannot be 0, because then it wouldn't be a 4-digit number. So we have three ways to choose where the 0 goes. The rest of the digits must not be zeros, and so for each remaining digit there would be 9 options. Then the answer is $3 \cdot 9^3 = \boxed{2187}$.

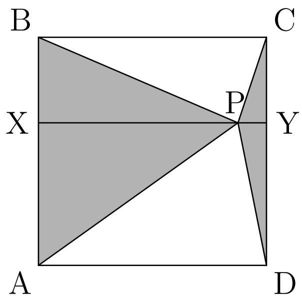
12. When Charles was born, suppose Allison was $2x$ years old and Bob was x years old. Then y years later coming to the present, Charles is y years old, Bob is $x + y$ years old, and Allison is $2x + y$ years old. The sum of the three ages currently is given to be 42, so

$$y + (x + y) + (2x + y) = 3x + 3y = 42 \implies x + y = 14.$$

But this is precisely Bob's age, $x + y$, so the answer is $\boxed{14}$.

13. There are two solutions to this problem.

Solution 1: Let a horizontal line through P meet AB and CD at points X and Y, as shown.



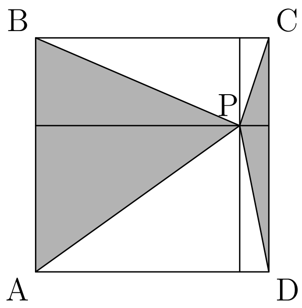
Setting base AB and height XP in $\triangle ABP$, and also setting base CD and height PY in $\triangle PCD$, the sum of their areas would be

$$\frac{1}{2}(AB)(XP) + \frac{1}{2}(CD)(PY).$$

But $AB = CD = 12$ and $XP + PY = BC = 12$ due to the given side length of the square, so the answer is

$$\frac{1}{2}(AB)(XP) + \frac{1}{2}(CD)(PY) = 6(XP + PY) = \boxed{72}.$$

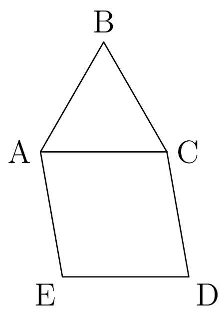
Solution 2: Draw vertical and horizontal lines through P, as shown.



The square would be broken up into four rectangular regions. But half of each rectangle would be shaded due to the diagonals. Summing all of these reveals that the total shaded area is actually just half of the square's area, so the answer would quickly become $\frac{1}{2}(12)^2 = \boxed{72}$.

14. Suppose the positive integer is $10a + b$, where a and b are digits. We are given that $10a + b = ab + a + b$, so simplifying gives $9a = ab$. However, because a is nonzero (it is given that they are two-digit positive integers), then we can divide both sides by a to see that $b = 9$. This is the only condition required. Then our such numbers would be $19, 29, 39, \cdots, 99$. The average of the nine of them is 59, so since there are 9 terms, our answer is $59 \cdot 9 = \boxed{531}$.

15. Draw the diagram with AC, as shown.



Because all sides in pentagon $ABCDE$ are equal, then $AB = BC$, implying that $\angle BCA = \angle CBA$. From $\angle ABC = 60°$, they must be equal at $60°$, so therefore $\triangle ABC$ is equilateral. Then $AB = BC = AC$, so $AC = CD = DE = EA$, implying that $ACDE$ is a rhombus. Because we are given that $\angle EAB = 140°$, then $\angle EAC = 80°$, so it follows that $\angle ACD = 100°$. But we know $\angle ACB = 60°$, so we conclude that $\angle BCD = 60° + 100° = 160°$, giving an answer of $\boxed{160}$.

16. Note that the probability of the second roll being greater than the first roll is equal to the probability that the first roll is greater than the second roll, due to symmetry. But any roll can either have the first roll greater, the second roll greater, or both equal. The probability that both are equal is $\frac{6}{36} = \frac{1}{6}$ because there are 6 ways to be equal, so the answer is $\frac{1}{2}\left(1 - \frac{1}{6}\right) = \boxed{\frac{5}{12}}$.

17. Suppose the ant started at 0. After moving 6 times, we are trying to find the probability that it is at either 2 or -2. Due to symmetry, we can just consider the probability that it goes to 2, then multiply the probability for that by 2. This can only be done with 4 right steps and 2 left steps, so we are trying to count the number of ways to arrange them. But that count would just be $\binom{6}{2} = 15$. The total number of possible paths the ant could go is determined by which direction it goes for each of the six steps, which is 2^6. So the probability it goes to 2 is $\frac{15}{64}$, and so the answer to the problem is $\frac{15}{64} \cdot 2 = \boxed{\frac{15}{32}}$.

18. Having identical unit digits in base 16 is equivalent to stating that

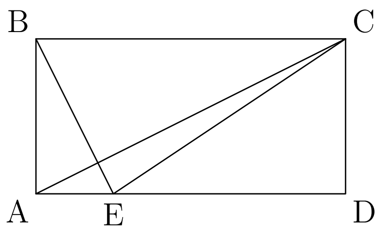
$$p^3 \equiv p^2 \pmod{16} \implies p^2(p-1) \equiv 0 \pmod{16}.$$

But recall p was given to be prime. So if $p = 2$ then $p^2 = 4$, which would not satisfy that equivalence relation. So p would have to be odd, meaning that $p^2 \not\equiv 0 \pmod{16}$. Thus division by p^2 would reveal

$$p - 1 \equiv 0 \pmod{16} \implies p \equiv 1 \pmod{16}.$$

So the possible values of p less than 100 would be $17, 33, 49, 65, 81, 97$ by listing them out. The only two that are actually prime are 17 and 97, and the sum of these two would be our answer, $\boxed{114}$.

19. We can draw a diagram, like as shown below.



Suppose that we set $AE = x$, so $ED = 3x$ and then $BC = AD = 4x$. But we can actually do a little angle-chasing. Because $AC \perp BE$, then $\angle AEB = 90 - \angle CAE$. But we know that $\angle BAE = 90°$, so $\angle BAC = 90 - \angle CAE = \angle AEB$. But combined with $\angle BAC = \angle CBA = 90°$, we would get the two similar triangles $\triangle ABE \sim \triangle BCA$ by AA Similarity. It follows by definition that $\frac{BC}{AB} = \frac{AB}{AE}$, so $AB^2 = (BC)(AE) = (4x)(x) = 4x^2$. Therefore, we can find that $AB = 2x$. Now, recall we are trying to find the value of the ratio $\frac{CE}{AE}$. By the Pythagorean Theorem, it can be found that $CE = \sqrt{(3x)^2 + (2x)^2} = x\sqrt{13}$, so the answer is $\frac{CE}{AE} = \frac{x\sqrt{13}}{x} = \boxed{\sqrt{13}}$.

20. Given that $mn = 1800$, $m - n = 600$, and the definitions of the functions, we are just finding the value of

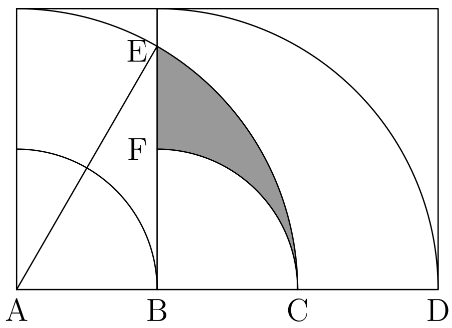
$$(1@(m\$n)) \cdot (1\$(m@n)) = \frac{1 - \frac{mn}{m-n}}{1 \cdot \frac{mn}{m-n}} \cdot \frac{1 \cdot \frac{m-n}{mn}}{1 - \frac{m-n}{mn}}.$$

This problem all boils down to simplification. We proceed by the logic below, multiplying both the numerator and denominator by some expression both fractions.

$$\begin{aligned}
&\frac{1 - \frac{mn}{m-n}}{1 \cdot \frac{mn}{m-n}} \cdot \frac{1 \cdot \frac{m-n}{mn}}{1 - \frac{m-n}{mn}} \\
=&\frac{m - n - mn}{mn} \cdot \frac{m-n}{mn - (m-n)} \\
=&\frac{m-n}{mn} \cdot \frac{m-n-mn}{mn - m + n} \\
=&\frac{n-m}{mn}.
\end{aligned}$$

Substituting -600 for $n - m$ and 1800 for mn, it follows that the answer is $\boxed{-\frac{1}{3}}$.

21. Consider the diagram below, with a few additional components drawn and labeled.



Recall we are given that $AB = BC = CD = 1$. Notice that the quarter circle with center A that passes through C has radius 2. Because $AE = AC = 2$ and $AB = 1$, then $\triangle AEB$ is a 30-60-90 triangle. Then the area of sector AEC is just $\frac{1}{6} \cdot 2^2 \cdot \pi = \frac{2}{3}\pi$. However, to get the desired region, we can just subtract quarter circle BFC and triangle AEB from sector AEC. The area of $\triangle AEB$ is $\frac{1}{2} \cdot 1 \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$, and the area of the quarter circle BFC is $\frac{1}{4} \cdot 1^2 \cdot \pi = \frac{1}{4}\pi$. Therefore, the answer is

$$\frac{2}{3}\pi - \frac{\sqrt{3}}{2} - \frac{1}{4}\pi = \frac{5}{12}\pi - \frac{\sqrt{3}}{2} = \boxed{\frac{5\pi - 6\sqrt{3}}{12}}.$$

22. Observe that we can just consider the As and the Ds on their own before adding back in the Bs and Cs. Considering all $\frac{6!}{3!3!} = 20$ permutations of $AAADDD$, only one has the rightmost A to the left of the leftmost D, which is that one.

So across all permutations of $AAABBBCCCDDD$, the probability that the rightmost A is to the left of the leftmost D is $\frac{1}{20}$. The total number of permutations without restrictions could be counted by choosing the 3 As first, the 3 Bs next, and so on. Then using our earlier derived probability, the number of desired permutations is

$$\frac{1}{20} \cdot \binom{12}{3}\binom{9}{3}\binom{6}{3}\binom{3}{3} = \frac{1}{20} \cdot 220 \cdot 84 \cdot 20 \cdot 1 = 220 \cdot 84 = \boxed{18480}.$$

Alternate Solution: After observing that the As and the Ds must come in that order, we can essentially insert the remaining letters into the expression. We can first find the 6 "spots" that the Bs and the Cs will take, before choosing which 3 spots will be which letter.

But taking this in another direction, this is equivalent to just choosing the 6 spots where the Bs and Cs go in a given string of length 12. Then there are $\binom{6}{3}$ ways to choose which 3 will be Bs, from which the Cs would follow. Then the answer would follow again as

$$\binom{12}{6}\binom{6}{3} = 924 \cdot 20 = \boxed{18480}.$$

23. Suppose that $n! + 3 = m^2$ for some positive integers m and n. Analyzing modulo 4, we can observe that

$$n! + 3 = m^2 \equiv 0, 1 \pmod 4 \implies n! \equiv 1, 2 \pmod 4.$$

The key observation here is to realize that for $n \geq 4$, $n!$ must be divisible by 4. Because perfect squares only allowed us modulo 4 residues of 1 and 2, it follows that there would exist no such m for when $n!$ is a multiple of 4. Then no $n \geq 4$ would ever make a perfect square.

It remains to check each case. For $n = 1$, it would equal 4, which works. Then $n = 2$ gives 7, which doesn't work, and $n = 3$ gives us 9, which does work. The sum of all positive integers n for which $n! + 3$ is a square is our desired answer, which would be $1 + 3 = \boxed{4}$.

24. For the fractional parts of x^2 and x to be identical, this would imply that $x^2 = x + k$ for some integer k. Rearranging the terms gives $x^2 - x - k = 0$, and applying the quadratic formula for x gives

$$x = \frac{1 \pm \sqrt{1+4k}}{2}.$$

We want to find the smallest positive value of x. Clearly we would need k to be nonnegative for x to be real. If $k = 0$, then our two roots would be 0 and 1, which are integers so we discard them. But if we let $k = 1$, then observe that while one root is negative, the other one works. Specifically, we would obtain our positive real value for x to be

$$x = \frac{1+\sqrt{5}}{2}.$$

For higher k, the $+$ case in the $\pm$ sign would only grow larger, and the $-$ case for the $\pm$ sign would become more and more negative. So this must be the smallest positive real value that satisfies the conditions. Then the value of x^2 can be computed, so

$$\{x^2 + x\} = \left\{\frac{3+\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2}\right\} = \{2+\sqrt{5}\}.$$

Observe that $2 < \sqrt{5} < 3$ by comparing their squares, so it follows that $\{x^2 + x\} = \{2+\sqrt{5}\} = \boxed{\sqrt{5}-2}$.

25. Firstly prime factorize

$$2025^{2026} = (3^4 \cdot 5^2)^{2026} = 3^{8104} \cdot 5^{4052}.$$

The main idea here is that we can consider the factors of 3 and the factors of 5 separately from each other, just looking at their exponents. Focus on the prime factor of 3 first. We randomly select two integers $0 \le e_1, e_2 \le 8104$ and find the probability that $e_1 + e_2 \le 8104$.

But then notice the probability that $e_1 + e_2 < 8104$ is the same as the probability that $e_1 + e_2 > 8104$, because of symmetry about 8104 - they can sum from 0 to 16208, and the number of ways to get a sum of k is the same as the number of ways to get a sum of $2 \cdot 8104 - k$ by considering $8104 - e_1$ and $8104 - e_2$ for any e_1 and e_2 with $e_1 + e_2 = k$.

Now, the probability that $e_1 + e_2 = 8104$ is $\frac{1}{8105}$, since for any choice of e_1 from 0 to 8104, there is exactly one number e_2 from 0 to 8104 that would make their sum 8104. If we subtracted this from 1 and divided the result by 2, it would be equivalent to the probability that $e_1 + e_2 < 8104$. So our probability here with $e_1 + e_2 \le 8104$ would become

$$\frac{1}{2}\left(1 - \frac{1}{8105}\right) + \frac{1}{8105} = \frac{1}{2}\left(1 + \frac{1}{8105}\right) = \frac{4053}{8105}.$$

Similarly, the factor of 5 would have symmetry about 4052, giving a probability of

$$\frac{1}{2}\left(1 + \frac{1}{4053}\right) = \frac{2027}{4053}.$$

Finally, the probability that $d_1 d_2$ is a divisor of 2025^{2026} would be the product of these, making the answer

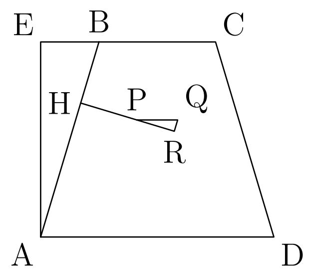
$$\frac{4053}{8105} \cdot \frac{2027}{4053} = \boxed{\frac{2027}{8105}}.$$

26. Any element in $\mathcal{S}$ can be written in the form $\frac{2026-n}{n}$, where n is a positive integer $1 \le n \le 2025$, which simplifies to $\frac{2026}{n} - 1$. Then we can observe that any fraction which simplifies has the property that n is not relatively prime to 2026, with the only exception being $n = 1$. Then it remains to count the number of positive integers n such that $\gcd(n, 2026) = 1$, and subtract 1 to get our answer.

From an initial value of 2026, there are $\frac{2026}{2} = 1013$ of them which are divisible by 2 and $\frac{2026}{1013} = 2$ of them divisible by 1013. But if we just subtract them from 2026, this double-subtracts the multiples of 2026, so since there is $\frac{2026}{2026} = 1$ of them, then by the Principle of Inclusion-Exclusion, our count is $2026 - 1013 - 2 + 1 = 1012$. Then subtracting 1 for $n = 1$ gets us our answer, $\boxed{1011}$.

Remark: The Euler phi function, $\phi(n)$, is also very applicable here. The number of values of n such that $\gcd(n, 2026) = 1$ for $1 \le n \le 2026$ is simply just $\phi(2026) = 2026\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{1013}\right) = 1012$, so we can subtract 1 to get the same result.

27. Let us construct a new diagram with some altitudes drawn, as shown below.



Here, H is the foot of the altitude from P on AB, E is the foot of the altitude from A to BC, and R is the foot of the altitude from Q to HP. Observe that $HR \perp AB$ and $HR \perp RQ$ implies $AB \parallel QR$, and given the same "heights" to the bases that P and Q shared, it follows that $PQ \parallel BC$.

From these two connections, we can deduce that $\angle PQR = \angle ABE$. But coupled with $\angle AEB = \angle PRQ = 90°$, it follows that $\triangle AEB \sim \triangle PRQ$ by AA similarity. Now, because the heights from both P and Q to BC and AD were 4 and 6, respectively, $AE = 4 + 6 = 10$. But $HP = 3$ is given, and we know that $HR = 5$ due to the height from Q to AB being given as 5. Thus $PR = 2$, so applying the similarity gives

$$\frac{AB}{PQ} = \frac{AE}{PR} = \frac{10}{2} = \boxed{5}.$$

28. We can shift the index in our definition

$$x_n = \sum_{k=1}^{n-1} kx_{n-k}$$

so that the summation can be simplified. Namely, using $n-1$ rather than n gives

$$x_{n-1} = \sum_{k=1}^{n-2} kx_{n-1-k} = \sum_{k=2}^{n-1} (k-1)x_{n-k}.$$

Subtracting these two definitions gives us

$$x_n - x_{n-1} = \sum_{k=1}^{n-1} (kx_{n-k}) - \sum_{k=2}^{n-1} ((k-1)x_{n-k}).$$

But we can "subtract" $(1-1)x_{n-1}$ from the RHS because it equals 0, transforming the expression into

$$\sum_{k=1}^{n-1}(kx_{n-k}) - \sum_{k=2}^{n-1}((k-1)x_{n-k}) - (1-1)x_{n-1} = \sum_{k=1}^{n-1}(kx_{n-k}) - \sum_{k=1}^{n-1}((k-1)x_{n-k}) = \sum_{k=1}^{n-1} x_{n-k}.$$

From our work above we have derived an important relationship,

$$x_n - x_{n-1} = \sum_{k=1}^{n-1} x_{n-k} = \sum_{k=1}^{n-1} x_k.$$

But we can shift the indices again to get the summations to simplify even further. Observe that

$$x_{n-1} - x_{n-2} = \sum_{k=1}^{n-2} x_k,$$

so if we subtract these two equations, we would get

$$(x_n - x_{n-1}) - (x_{n-1} - x_{n-2}) = \sum_{k=1}^{n-1} x_k - \sum_{k=1}^{n-2} x_k = x_{n-1}.$$

But now we have just derived an extremely useful closed form relationship. For large enough n where the summations could be simplified above in those ways, then we must have that

$$x_n - 2x_{n-1} + x_{n-2} = x_{n-1} \implies x_n + x_{n-2} = 3x_{n-1} \implies \frac{x_n + x_{n-2}}{x_{n-1}} = 3.$$

But this is precisely what is asked in the problem if we set $n = 2027$, which is clearly large enough. So therefore our answer would just be

$$\frac{x_{2025} + x_{2027}}{x_{2026}} = \boxed{3}.$$

29. Let e_n denote the number of possible inputs assuming $f_n(x)$ equals a given even positive integer, and let o_n denote the number of possible inputs assuming $f_n(x)$ equals a given odd positive integer.

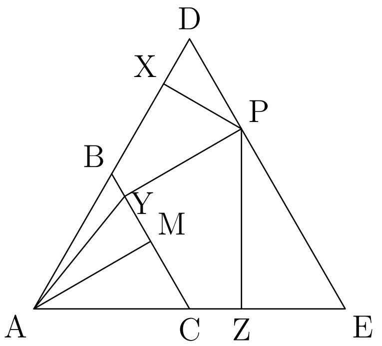
Consider what would happen to the number that goes into the last machine. If the output is odd, then we could either have added 1 or divided by 2. However, the $+1$ cannot happen as the initial even number (from even $+1 =$ odd) should've been divided by 2 instead. Thus we get one equation for the recursion $o_n = e_{n-1}$.

Additionally, if the output is even, then both options for the input are always possible with adding 1 to an odd or dividing an even by 2, giving the second equation $e_n = o_{n-1} + e_{n-1}$.

Note that 2026 is even, so we will solve the even-case recursion. Substituting $o_{n-1} = e_{n-2}$ gives the equation $e_n = e_{n-1} + e_{n-2}$, which is a Fibonacci-like recursion. We have that $e_0 = 1$ and $e_1 = 2$, so this is actually just the Fibonacci sequence with the indices shifted down 2 lower. Then e_{10} is the 12th Fibonacci number, which can be computed as $\boxed{144}$.

Remark: This is the third year in a row that this process/machine/function has shown up (2024 A22, 2025 A15/B27, 2026 A29). Maybe it could find another way to become a problem for the 4th year too...

30. Construct a new equilateral triangle $\triangle ADE$, as shown, where DE passes through P and $DE \parallel BC$.



From $XP = 2$ and $PZ = 4$, then we can evaluate that $DP = \frac{2}{\sqrt{3}} \cdot 2 = \frac{4\sqrt{3}}{3}$ and $PE = \frac{2}{\sqrt{3}} \cdot 4 = \frac{8\sqrt{3}}{3}$ making the side length of $\triangle ADE$ be $\frac{4\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} = 4\sqrt{3}$. But if we also draw an altitude from A into $\triangle ABC$, like shown here to M, we can figure out this length too. Specifically, AM and PY sum to $\triangle ADE$'s height, which would be $\frac{\sqrt{3}}{2} \cdot 4\sqrt{3} = 6$. But $PY = 3$, so it follows that the height in $\triangle ABC$ is $AM = 3$. This would make its side length equal to $2\sqrt{3}$ as a result.

Now, to find AY, it remains to find YM. To do this, notice the similar triangles $\triangle XDP \sim \triangle ZEP$ by AA similarity. Because $XP = 2$ and $PZ = 4$, then $\frac{DP}{PE} = \frac{1}{2}$ so we must have $PE = \frac{2}{3}DE = \frac{8\sqrt{3}}{3}$. Then using 30-60-90 triangle properties, it follows that

$$ZE = \frac{4\sqrt{3}}{3} \implies CZ = CE - ZE = 2\sqrt{3} - \frac{4\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}.$$

Now, look at how $\angle PYC = \angle PZC = 90°$, which suggests to use the Pythagorean Theorem. Specifically, drawing PC would give us

$$PZ^2 + CZ^2 = PY^2 + CY^2 \implies YC^2 = 4^2 + \left(\frac{2\sqrt{3}}{3}\right)^2 - 3^2 = 7 + \frac{4}{3} = \frac{25}{3}.$$

This would mean that $YC = \frac{5\sqrt{3}}{3}$, so since $MC = \sqrt{3}$, it follows that $YM = \frac{2\sqrt{3}}{3}$. Therefore, now that we have YM, we can apply the Pythagorean Theorem in $\triangle AYM$. Then our answer would be

$$AY^2 = 32 + \left(\frac{2\sqrt{3}}{3}\right)^2 = 9 + \frac{4}{3} = \boxed{\frac{31}{3}}.$$

Problem Credits

Problems 1-6 and 8-30 - Ethan Zhang, 29 problems.

Problem 7 - Andrew Luo, 1 problem.