2026 Clover Math Competition

Form A

May 1, 2026

1. A printer is loaded with 2026 pieces of paper. Katelyn needs to print out multiple copies of a document through this printer. One copy of the document requires 45 pieces of paper. What is the greatest number of full copies of the document that can be made?

2. Anthony has an unfair coin with a $\frac{1}{3}$ probability of landing heads and a $\frac{2}{3}$ probability of landing tails. What is the probability that when the coin is flipped two times, Anthony will get two tails?

3. What positive integer is closest to the value of the expression $\sqrt{5\pi}$?

4. Compute the greatest common divisor of 20260 and 20266.

5. A laptop costs 50% more than a phone. Then how many phones would have a combined cost that is 60% more than the combined cost of 15 laptops?

6. In an isosceles triangle, one angle is exactly 120° larger than another angle. Find the sum of all possible degree measures of the larger angle.

7. In a certain kingdom, everyone is either a knight, who tells the truth all the time, a knave, who lies all the time, or a joker, who gives a response at random. Betty makes the statement, "I am a knave." Find the sum of all the numbers below whose corresponding statement can possibly be true.

   (1) Betty is a knight.
   (2) Betty is a knave.
   (4) Betty is a joker.

   For example, if you think that Betty could be either a knight or a joker, your answer should be $1 + 4 = 5$. If none can be true, answer 0.

8. Abel, Beth, Charlie, and Darelyn each have a house on the same street, though not necessarily in this order.

   - Abel is 4 houses away from Beth
   - Beth is 7 houses away from Charlie
   - Charlie is 5 houses away from Darelyn
   - Darelyn is 8 houses away from Abel.

   How many houses away from Abel from Charlie?

9. Find the least even positive integer for which the remainder when it is divided by 3 is 1 and the remainder when it is divided by 5 is 3.

10. In the first half of a game, Timmy was thrown 12 balls, and he caught 25% of them. In the second half of the game, Timmy caught exactly 50% of the balls thrown at him, and in total he caught exactly 44% of the balls thrown at him. How many balls did he catch in total?

11. The number 2026 has exactly one zero in its digits. How many four digit numbers, including 2026, have exactly one digit equal to 0?

12. Allison, Bob, and Charles are three siblings in a family, and the sum of their ages in years is equal to 42. When Charles was born, Allison was exactly twice as old as Bob. How old is Bob currently, in years?

13. Point P lies inside square $ABCD$ of side length 12. Given that $AP = 13$, $CP = 5$, and $BP > DP$, then find the sum of the areas of $\triangle ABP$ and $\triangle CDP$.

14. Find the sum of all two-digit positive integers with the property that they are equal to the result when the product of their digits is added to the sum of their digits. For example, 29 satisfies this property because it is equal to $(2 \cdot 9) + (2 + 9)$.

15. In pentagon $ABCDE$, all sides are equal in length, $\angle EAB = 140°$, and $\angle ABC = 60°$. What is the degree measure of $\angle BCD$?

16. Sasha rolls a fair die twice, and writes down the two numbers in order. What is the probability that the second roll is less than the first roll?

17. Antler the ant is on an infinite number line. Every ten seconds, it randomly chooses left or right and moves one inch in that direction. After it has moved six times, what is the probability that it is exactly two inches from where it started?

18. Let p be a prime number with the property that, when written in base 16, p^3 and p^2 have the same unit digits. Find the sum of all possible values of p less than 100.

19. In rectangle $ABCD$ with $AB < BC$, point E lies on AD such that BE is perpendicular to AC. Given that $\frac{DE}{AE} = 3$, compute the value of $\frac{CE}{AE}$.

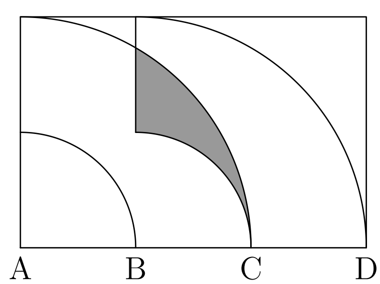
20. Define the two operations

$$a@b = \frac{a-b}{ab} \text{ and } a\$b = \frac{ab}{a-b}.$$

Let m and n be constants with the property that $mn = 1800$ and m is 600 greater than n. What is the value of the expression

$$(1@(m\$n)) \cdot (1\$(m@n))?$$

21. In the diagram shown, 2 windshield wipers are centered at A and B on a perfectly rectangular windshield and rotate 90°. Given that $AB = BC = CD = 1$ foot, then in square feet, what is the area of the shaded overlapping region?



22. How many permutations of $AAABBBCCCDDD$ have the rightmost A to the left of the leftmost D? (For example, the permutation $ABBACADDCBCD$ would work, whereas $AADADDBCBCBC$ would not satisfy that condition.)

23. Find the sum of all positive integers n for which $n! + 3$ is a perfect square.
(Recall that $n!$ denotes the product of the first n positive integers, e.g. $2! = 1 \cdot 2 = 2$.)

24. Let x be the smallest positive real number that isn't an integer for which

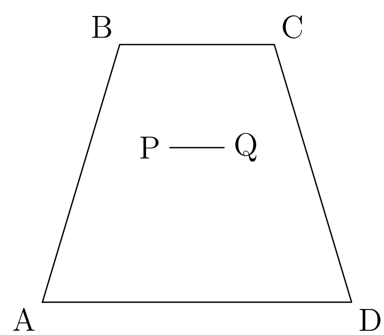
$$\{x^2\} = \{x\}.$$

Compute the value of the expression $\{x^2 + x\}$.
(Note that $\{x\}$ denotes the fractional part of x, e.g. $\{1.3\} = 0.3$ and $\{\pi\} = \pi - 3$.)

25. Let d_1 and d_2 be two (not necessarily distinct) randomly selected divisors of 2025^{2026}. What is the probability that $d_1 d_2$ is a divisor of 2025^{2026}?

26. Suppose $\mathcal{S}$ is the set of all positive rational fractions in lowest terms with the property that the numerator and denominator sum to 2026. How many elements are in $\mathcal{S}$? (Note that the fraction $\frac{2025}{1}$ is not an element, since it is not in lowest terms.)

27. Let P and Q be points in isosceles trapezoid $ABCD$ with $BC \parallel AD$ and $AB = CD$. The distances from P to AB, BC, CD, and DA are 3, 4, 5, and 6, respectively, and the distances from Q to AB, BC, CD, and DA are 5, 4, 3, and 6, respectively. Find $\frac{AB}{PQ}$.



28. Define a sequence x_n by $x_1 = 1$, and for all positive integers $n \geq 2$,

$$x_n = \sum_{k=1}^{n-1} k x_{n-k}.$$

What is the value of the expression

$$\frac{x_{2025} + x_{2027}}{x_{2026}}?$$

29. Let f be the function defined by

$$f_1(x) = \begin{cases} \frac{1}{2}x & \text{if } x \text{ is even} \\ x+1 & \text{if } x \text{ is odd} \end{cases}$$

and $f_{n+1}(x) = f_1(f_n(x))$ for all positive integers n. How many positive integers p satisfy

$$f_{10}(p) = 2026?$$

30. Let $\triangle ABC$ be an equilateral triangle. Point P lies outside the triangle on the opposite side of BC from A. Let X, Y, and Z be the projections of P onto AB, BC, and AC, respectively. Suppose that $PX = 2$, $PY = 3$, and $PZ = 4$. Compute AY^2.