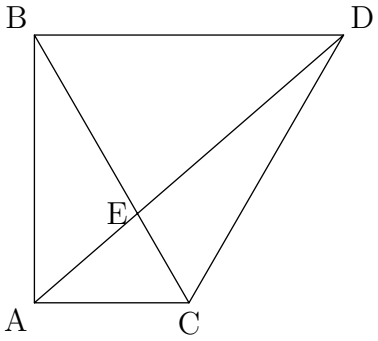
# 2025 Clover Math Competition

## Form A

1. Allison has 12 more clovers than Brian. After Allison gives Brian some number of his clovers, they each have an equal number of clovers. How many clovers did Allison give Brian?
2. What is the smallest two-digit prime whose digits also sum to a prime?
3. Tommy can run 100 meters in 20 seconds, and Billy can run 100 meters in 25 seconds. When Tommy and Billy run a 100m race, how many meters is Billy losing by when Tommy reaches the finish line?
4. In a field of 500 clovers, 102 are 4-leaf and the rest are 3-leaf. How many clovers does Norbit need to pick to guarantee he will have picked at least one 4-leaf clover?
5. In a recent survey at Element Tree Elementary School, 62 kids played chess. It was found that 38 kids played basketball, 23 of whom didn't play chess. How many kids who played chess didn't play basketball?
6. At a party, $\frac{2}{3}$ of the people are wearing a costume and $\frac{5}{7}$ of the people are eating snacks. What is the minimum fraction of people in the room wearing a costume and eating snacks?
7. A magic square has the property that the sum of the three numbers in every row, column, and diagonal is the same. What is the number in the square shown with a question mark?

| | | |
|---|---|---|
| 3 | 5 | |
| | | 2 |
| | | ? |

8. How many three-digit positive integers have the property that exactly two of the digits are the same?
9. Let $a_n$ be a recursive sequence with $a_1 = -2$ and $a_{n+1} = a_n^2 - 7$ for $n \geq 1$. Find $a_{2025}$.
10. In the diagram shown, $AC = 1$, $\angle ABC = 30°$, $\angle BAC = 90°$, and $\triangle BDC$ is equilateral. Find $BE$.




11. The point $(-6, 33)$ is reflected over the line $y = mx + b$ in the coordinate plane to $(8, 1)$, where $m$ and $b$ are constants. Find the value of $m + b$.
12. Negative numbers $x$ and $y$ satisfy the equation $x^2 + 4y - y^2 = 8x - 12$. What is $x - y$?

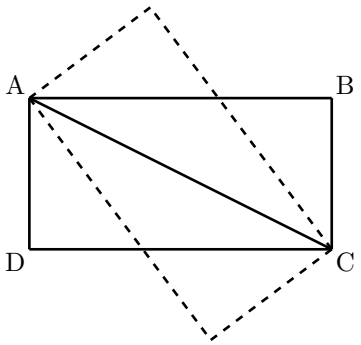
13. Suppose 50 initially empty hotel rooms are numbered 1-50. The rooms are filled and emptied according to the following process: every 2nd room is filled if empty and emptied if full, then every 3rd, then every 5th, then every 7th, and so on. At the end of this process, how many rooms are empty?

14. You're making a "clover chain" where each link is either a 3-leaf or a 4-leaf clover. You want to make a chain of 7 clovers total, with at least two 4-leaf clovers. How many possible "clover chain" combinations can be made?

15. A machine takes any positive integer as an input. If the input is even, then the machine divides it by 2. If the input is odd, then the machine adds 1. Three of these machines are chained together, so for example, if 6 is put into the first machine, they produce 2 as the output. Suppose Tommy puts in a number that results in a 5 as an output. What is the sum of all the possible numbers that Tommy could have put in?

16. Let $a$ and $b$ be positive integers such that $\frac{3}{8} < \frac{a}{b} < \frac{2}{5}$ and $a + b$ is minimized. What is $a + b$?

17. Rectangle $ABCD$ has side lengths $AB = 12$ and $BC = 6$. The rectangle is reflected across diagonal $AC$ to obtain a new rectangle. Find the area of the union of the two interiors of the rectangles.




18. Jonathan flips a coin six times in a row. What is the probability that the coin never lands on heads two times or more in a row?

19. Max computes $3^{2024}$ and writes the result on a chalkboard. He then adds up all of the number's digits, erases the original number, and replaces it with this new sum. He repeats this process until he is left with a single digit. What is this digit?

20. In a $5 \times 5$ grid of 3-leaf and 4-leaf clovers, where each square in the grid has exactly one clover, let $M$ be the minimum number of 4-leaf clovers needed so that no row, column, or main diagonal contains only 3-leaf clovers. Let $N$ be the number of ways to place these $M$ 4-leaf clovers so that no row, column, or main diagonal contains only 3-leaf clovers. Find $M + N$.

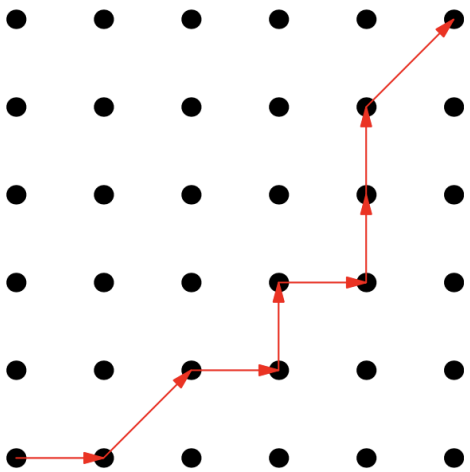
21. The circle with equation $(x - 2)^2 + (y - 4)^2 = c$ intersects the graph of $y = |x|$ at exactly 3 points. What is the product of all possible values of $c$?

22. Suppose $P(x) = x^4 - 2x^3 - 10x^2 + 5x - 1$ and let $Q(x)$ be the unique monic fourth degree polynomial with integer coefficients whose roots are double the negative reciprocals of the roots of $P(x)$. Find $Q(1)$.

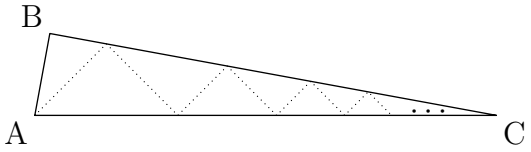
23. There are exactly two types of people in a certain group, regular and popular. All regular people have two friends in the group, and all popular people have five friends in the group. All popular people only have regular friends, but regular people can be friends with anyone. Of all pairs of people who are friends with each other, 80% of those pairs contain a popular person. Find the ratio of the number of popular people to the number of total people in this group.

24. Let $a_0, a_1, a_2, \ldots$ be a sequence of real numbers such that $a_0 = 1$, $a_1 = 2$, and for $n \geq 2$, $a_n a_{n-1} = a_0 + a_1 + \cdots + a_{n-1}$. Suppose that $a_0 + a_2 + a_4 + \cdots + a_{2024} = a_k$ for some positive integer $k$. Find the remainder when $k$ is divided by 1000.

25. In a $6 \times 6$ rectangular grid of lattice points, a bug starts at the bottom left corner. On any given step, the bug can move one step either up, right, or diagonally up and right. How many paths going from the bottom left corner to the upper right corner are exactly eight steps long? One such path is shown.



26. Let $\triangle ABC$ be a right triangle with $\angle ABC = 90°$, $AB = 1$, and $BC = 9$. Suppose that right isosceles triangles are inscribed with their bases on $AC$ and vertices on $BC$ and that any two consecutive inscribed triangles share a vertex.




Assuming the pattern goes infinitely, what is the sum of the lengths of all the dotted line segments which create the inscribed isosceles right triangles?

27. In tetrahedron $ABCD$, $AB = AC = AD = 5$ and $BC = CD = DB = 6$. A sphere is situated tangent to faces $ABC$, $ACD$, and $ADB$ such that its center lies on face $BCD$. What is the radius of this sphere?

28. Circles $O_1$ and $O_2$ are tangent to each other at point $P$ and have radii 2 and 1, respectively. Point $Q$ is on circle $O_1$ such that $PQ = 3$. Line $PQ$ intersects circle $O_2$ again at point $R$. Find the area of triangle with vertices $P, R$, and the center of circle $O_1$.

29. An ant is in the middle square in a $21 \times 21$ square grid. After every minute, it moves at random going one space up, down, left, or right such that it stays on the square grid. Then the probability that after 10 minutes, the ant is back on the middle square can be written as $\frac{N}{2^M}$ for integers $N$ and $M$ with $N$ odd. Find $N + M$.

30. The range of all values of $k$ such that the equation $x\{x\} = k\lfloor x \rfloor$ has exactly 2025 positive solutions for $x$ can be expressed as $\frac{p}{q} \leq k < \frac{r}{s}$, where $p$, $q$, $r$, and $s$ are positive integers with $\gcd(p, q) = \gcd(r, s) = 1$. Find the value of $p + q + r + s$.
($\lfloor x \rfloor$ denotes the greatest integer less than or equal to $x$, and $\{x\}$ denotes the decimal part of $x$.)